EXPONENTIALS AND LOGARITHMS   *Answers*

1   **a**   20.1   **b**   0.135   **c**   13.6   **d**   −0.598   **e**   1.97   **f**   0.434

2   **a** $= 4$ **b** $= e^{\ln 3} = 3$ **c** $= 2e^{\ln \frac{1}{6}} = \frac{1}{3}$ **d** $= 7$ **e** $= \ln e^{-1} = -1$ **f** $= -0.5$

3   **a** $x = 4$ **b** $x = 17$ **c** $x^2 = 25$ **d** $\frac{1}{x} = \frac{1}{3}$

                                                          $x > 0 \ \therefore \ x = 5$ $x = 3$

4   **a** $x = e^{15}$ **b** $\ln t = 6$ **c** $x - 4 = e^7$

                                      $t = e^6$ $x = e^7 + 4$

      d $\ln 5y = 8$ **e** $\frac{1}{2}x + 3 = e^{2.5}$ **f** $4 - 3x = e^{11}$

            $5y = e^8$ $\frac{1}{2}x = e^{2.5} - 3$ $3x = 4 - e^{11}$

            $y = \frac{1}{5}e^8$ $x = 2e^{2.5} - 6$ $x = \frac{1}{3}(4 - e^{11})$

5   **a** $x = \ln 0.7$ **b** $e^y = 2$ **c** $5x = \ln 3$

                                     $y = \ln 2$ $x = \frac{1}{5}\ln 3$

      d $4t + 1 = \ln 12$ **e** $e^{2x-3} = 14$ **f** $e^{4-5x} = \frac{7}{2}$

            $t = \frac{1}{4}(\ln 12 - 1)$ $2x - 3 = \ln 14$ $4 - 5x = \ln \frac{7}{2}$

                                      $x = \frac{1}{2}(\ln 14 + 3)$ $x = \frac{1}{5}(4 - \ln \frac{7}{2})$

6   **a** $e^x = 12$ **b** $15x - 7 = e^4$ **c** $e^{\frac{1}{2}y+3} = \frac{11}{4}$

            $x = \ln 12 = 2.48$ $x = \frac{1}{15}(e^4 + 7) = 4.11$ $\frac{1}{2}y + 3 = \ln \frac{11}{4}$

                                                         $y = 2(\ln \frac{11}{4} - 3) = -3.98$

      d $\ln (5 - 2x) = \frac{7}{3}$ **e** $10 - 3y = e^e$ **f** $2\ln x + 3\ln x = 19$

            $5 - 2x = e^{\frac{7}{3}}$ $y = \frac{1}{3}(10 - e^e) = -1.72$ $\ln x = \frac{19}{5}$

            $x = \frac{1}{2}(5 - e^{\frac{7}{3}}) = -2.66$ $x = e^{\frac{19}{5}} = 44.70$

      g $e^{\frac{9}{4}x} = 3$ **h** $e^{3t-1} = 4$ **i** $\ln \frac{2x-5}{x} = \frac{1}{4}$

            $\frac{9}{4}x = \ln 3$ $3t - 1 = \ln 4$ $2x - 5 = e^{\frac{1}{4}}x$

            $x = \frac{4}{9}\ln 3 = 0.49$ $t = \frac{1}{3}(\ln 4 + 1) = 0.80$ $(2 - e^{\frac{1}{4}})x = 5$

                                                         $x = \frac{5}{2 - e^{\frac{1}{4}}} = 6.98$

7 $2e^{2x} - 11e^x + 12 = 0$

      $(2e^x - 3)(e^x - 4) = 0$

      $e^x = \frac{3}{2}, 4$

      $x = \ln \frac{3}{2}, \ln 4$

8 **a** $= \dfrac{(3x-4)(x-2)}{(x-2)(x-3)} = \dfrac{3x-4}{x-3}$

9 $e^{5y} - x = 0 \qquad \Rightarrow \qquad 5y = \ln x$

b $\ln \dfrac{3x^2 - 10x + 8}{x^2 - 5x + 6} = \ln 2x$

$\ln x^4 = 7 - y \qquad \Rightarrow \qquad 4 \ln x = 7 - y$

$\dfrac{3x^2 - 10x + 8}{x^2 - 5x + 6} = 2x$

sub. $\qquad 20y = 7 - y$

$\dfrac{3x-4}{x-3} = 2x$

$y = \tfrac{1}{3}$

$3x - 4 = 2x(x-3)$
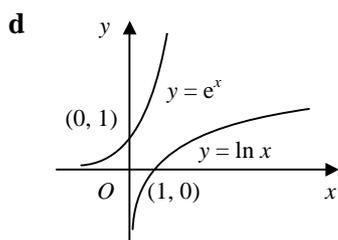$2x^2 - 9x + 4 = 0$
$(2x - 1)(x - 4) = 0$
$x = \tfrac{1}{2} , 4$

$\therefore \ x = e^{\frac{5}{3}} = 5.29, \ y = 0.33$

10 **a**

b

c

d

e

f

11 **a**
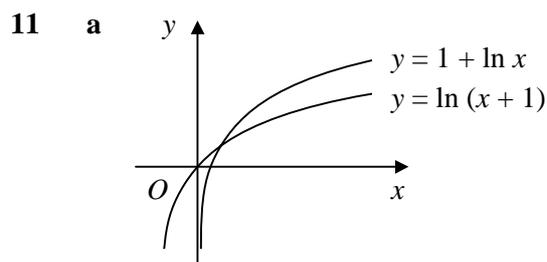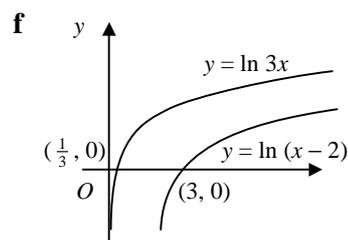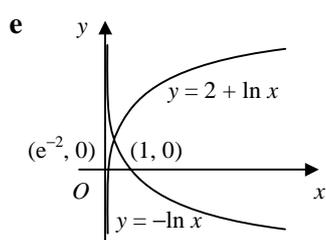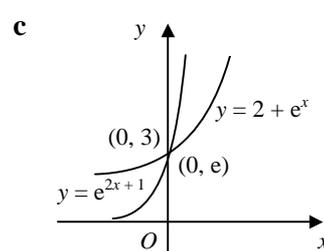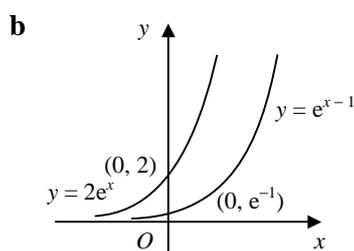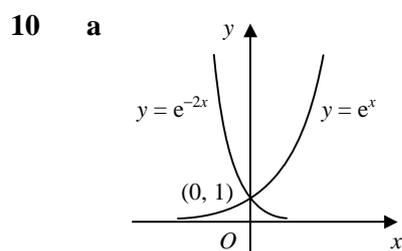
b $\ln (x + 1) = 1 + \ln x$
$\ln (x + 1) - \ln x = 1$
$\ln \dfrac{x+1}{x} = 1$
$\dfrac{x+1}{x} = e$
$x + 1 = ex$
$1 = x(e - 1)$
$x = \dfrac{1}{e-1}$

12 **a** 3

b $x = 0$ $\therefore$ $y = 3 + e^{-1}$
$\therefore$ $(0, 3 + e^{-1})$

c $3 + e^{2x-1} = 7$
$e^{2x-1} = 4$
$2x - 1 = \ln 4$
$x = \frac{1}{2}(1 + \ln 4)$
$x = \frac{1}{2} + \ln 2$

13 **a** $t = 10$, $N = 50e^{-2} = 6.77$ (3sf)

b $3 = 50e^{-0.2t}$
$t = -5 \ln \frac{3}{50} = 14.1$ (3sf)

14 **a** $160 = 240e^{180k}$
$k = \frac{1}{180} \ln \frac{2}{3} = -0.00225$ (3sf)

b $m = 240e^{-0.002253t}$
$120 = 240e^{-0.002253t}$
$t = \frac{-1}{0.002253} \ln \frac{1}{2} = 308$ years (3sf)

15 **a** $t = 15$, $N = 20e^{0.6} = 36.4$ (3sf)

b **i** $k = 20e^{0.04t}$

$t = \frac{\ln(\frac{k}{20})}{0.04} = 25 \ln \frac{k}{20}$

ii $2k = 20e^{0.04t}$

$t = \frac{\ln(\frac{k}{10})}{0.04} = 25 \ln \frac{k}{10}$

c time for N to increase from k to $2k$
$= 25 \ln \frac{k}{10} - 25 \ln \frac{k}{20}$

$= 25 \ln \frac{(\frac{k}{10})}{(\frac{k}{20})}$

$= 25 \ln 2$
$\therefore$ time for N to double is constant

16 **a** $300 = N_0 e^{10k}$ $\Rightarrow$ $N_0 = \frac{300}{e^{10k}}$

$225 = N_0 e^{20k}$

$\therefore$ $225 = \frac{300}{e^{10k}} \times e^{20k}$

$e^{10k} = \frac{3}{4}$

$k = \frac{1}{10} \ln \frac{3}{4} = -0.0288$ (3sf)

$N_0 = \frac{300}{\frac{3}{4}} = 400$

b $N = 400e^{-0.02877t}$
$150 = 400e^{-0.02877t}$
$t = \frac{-1}{0.02877} \ln \frac{3}{8} = 34.1$ (3sf)